Phys 31

Mechanics, Electricity & Magnetism

Vector Analysis

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VECTOR ANALYSIS

Physical Quantity

Scalar – specified by their magnitude only

– e.g. Time (t), temperature (T), Mass (m), distance (s), density (ρ)

Vector – specified by their magnitude and direction

– e.g. Force ($\vec{F}$), Electric field ($\vec{E}$), weight ($\vec{w}$), displacement ($\vec{s}$)

VECTOR ANALYSIS

Physical Quantity

Scalar → are added by ordinary algebraic method

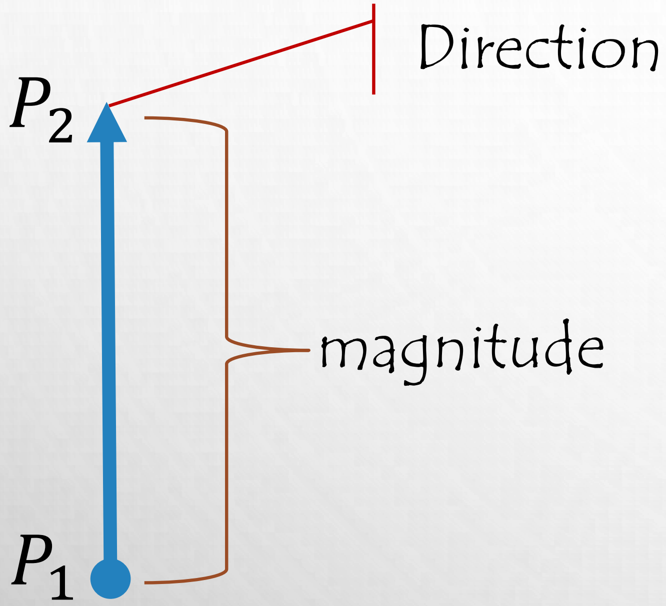
– specified by their magnitude only

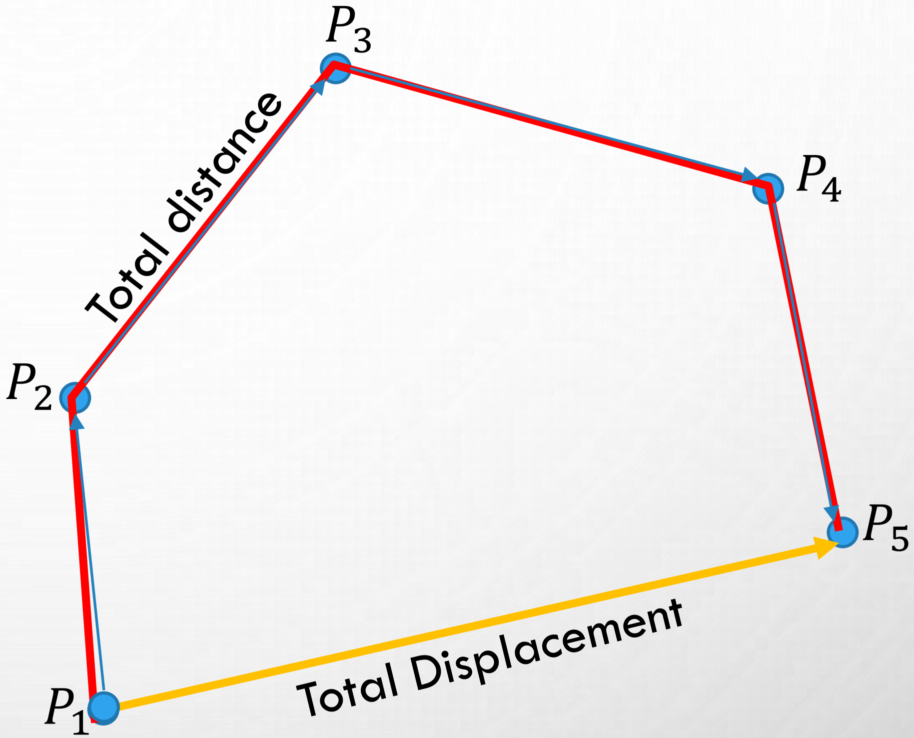
Vector → are added by geometric method

– specified by their magnitude and direction

VECTOR ANALYSIS

Vector addition or vector sum



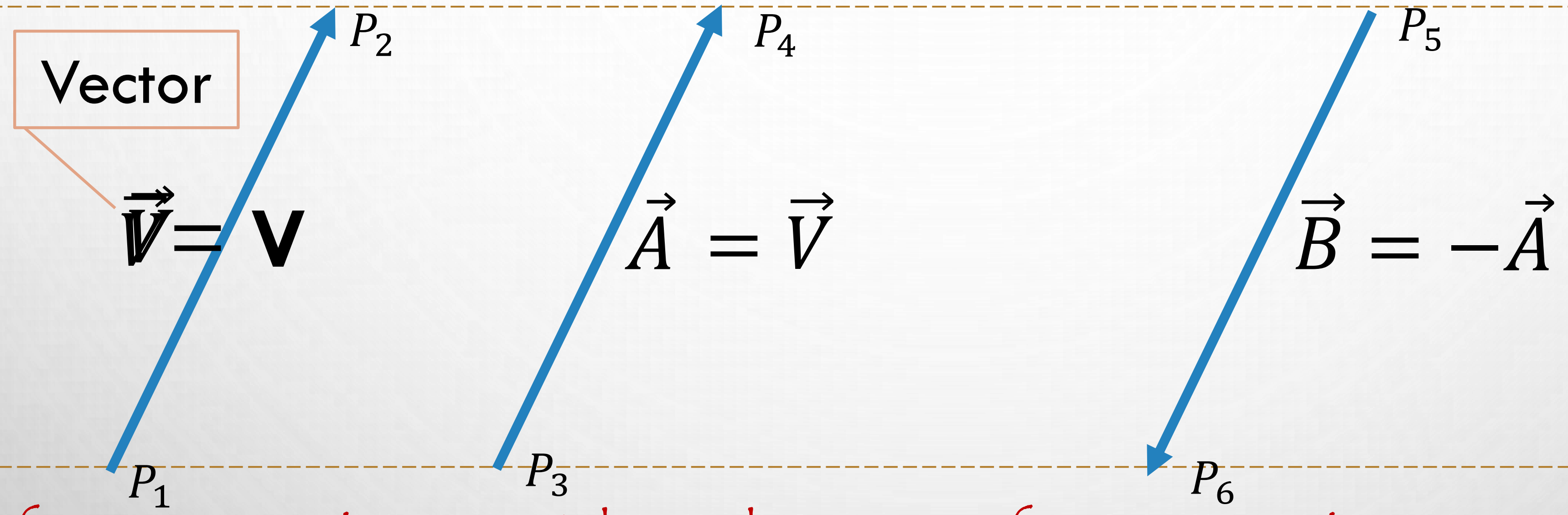


What is the total distance and total displacement from P_1 to P_5?

VECTOR ANALYSIS

Vector addition or vector sum

Note: The magnitude of vector $\vec{V}$ is $|\vec{V}|$ *or* V



If vector V and vector A have the same magnitude, then the two vectors are equal.

If vector B and vector A have the same magnitude but in opposite direction, then the two vectors are not equal.

VECTOR ANALYSIS

Methods of finding Vector sum or resultant of forces

- Graphical Method → Parallelogram Method; Polygon Method

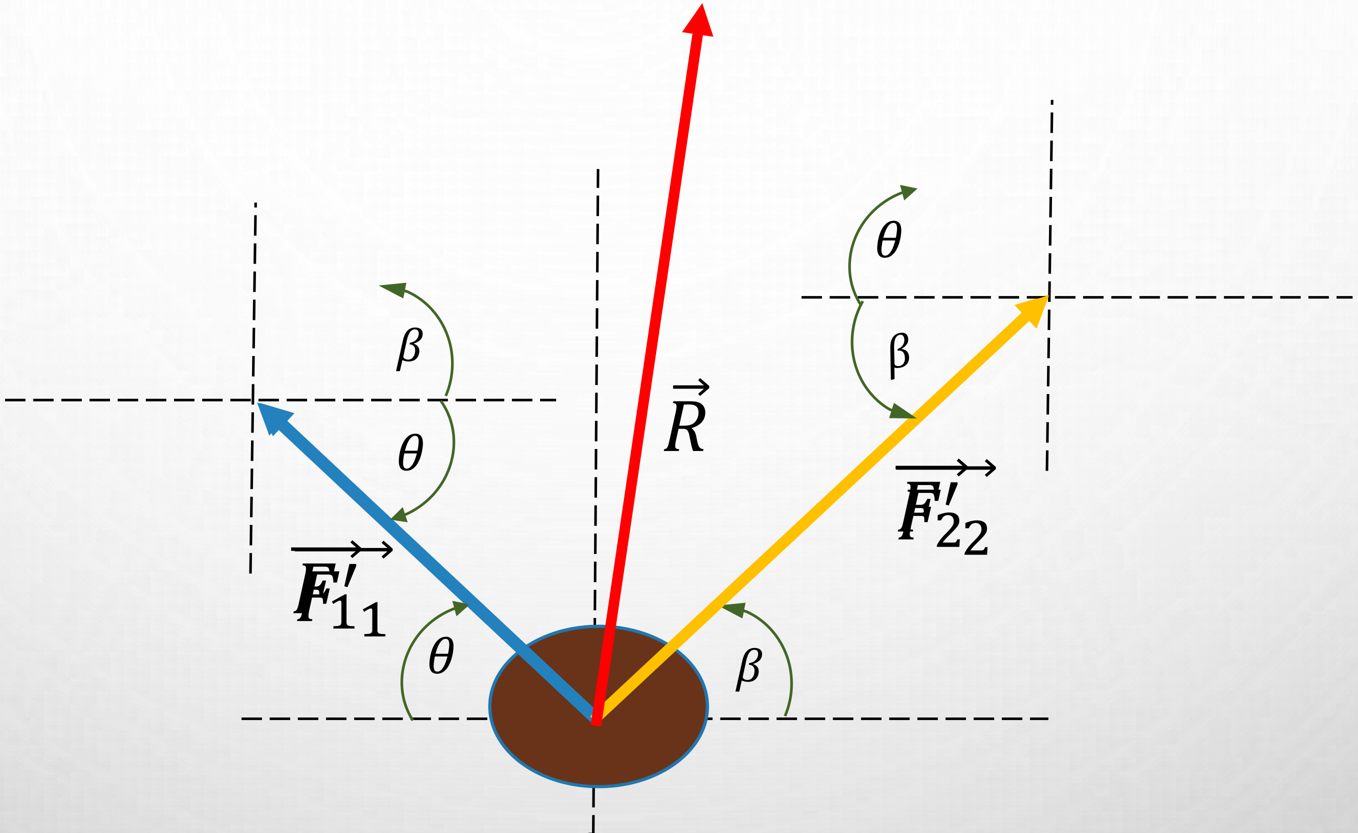
  Material needed:
  - Triangular scale or ruler
  - Protractor
  - Writing paper and pencil

- Analytical Method → Cosine law Method; Component Method

  Material needed:
  - calculator
  - Writing paper and pen

VECTOR ANALYSIS

- Graphical Method Parallelogram Method



Two forces

(Drawn to scale)

- **Graphical Method: Parallelogram Method**

Suppose Given:

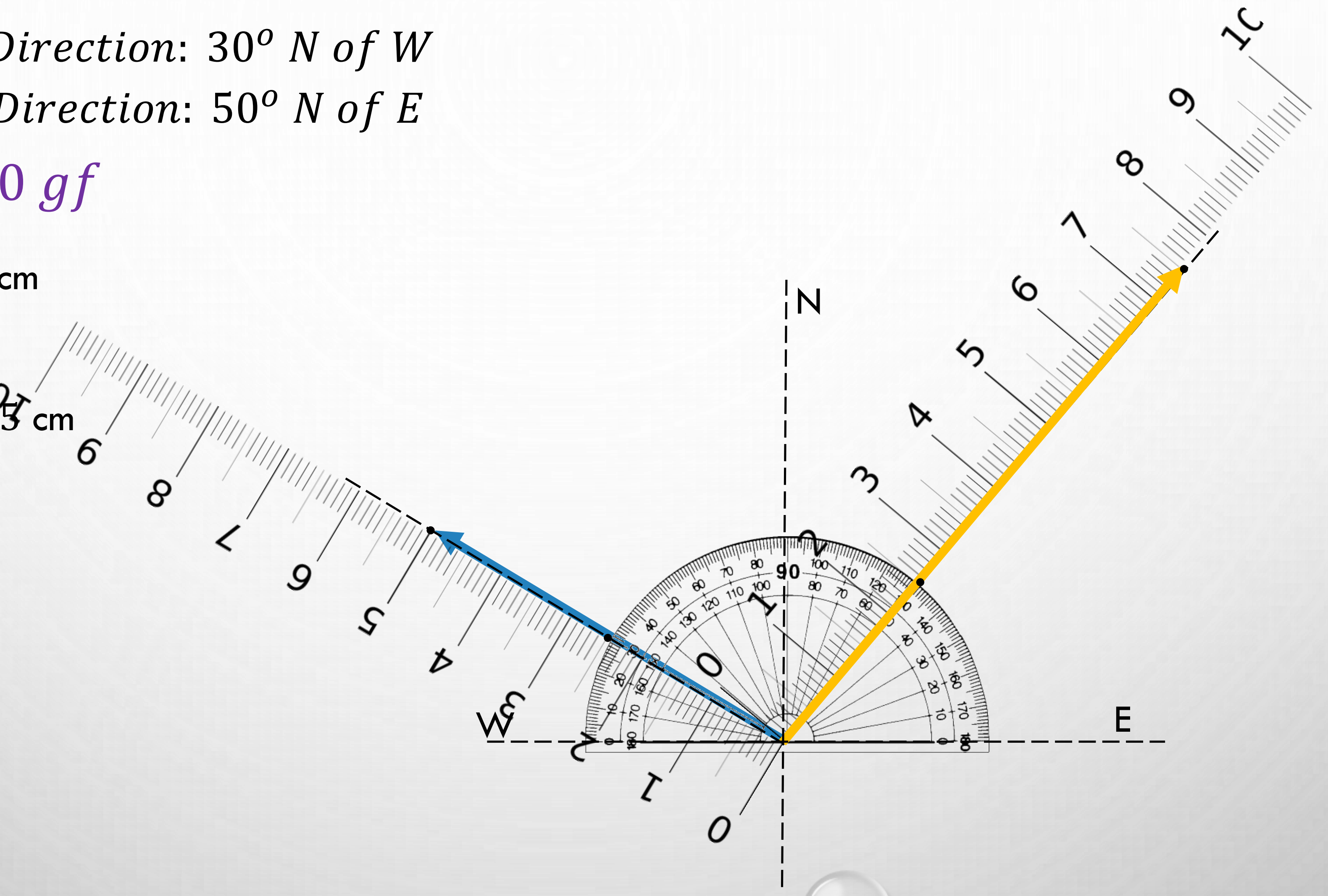
$\vec{F}_1 = 100\ gf$ Direction: $30^o\ N\ of\ W$

$\vec{F}_2 = 150\ gf$ Direction: $50^o\ N\ of\ E$

Scale $1cm : 20\ gf$

$$100\ gf \times \frac{1cm}{20\ gf} = 5\ cm$$

$$150\ gf \times \frac{1cm}{20\ gf} = 7.5\ cm$$



• Graphical Method: Parallelogram Method

Suppose Given:

$\vec{F}_1 = 100\ gf \quad Direction: 30^o\ N\ of\ W$

$\vec{F}_2 = 150\ gf \quad Direction: 50^o\ N\ of\ E$

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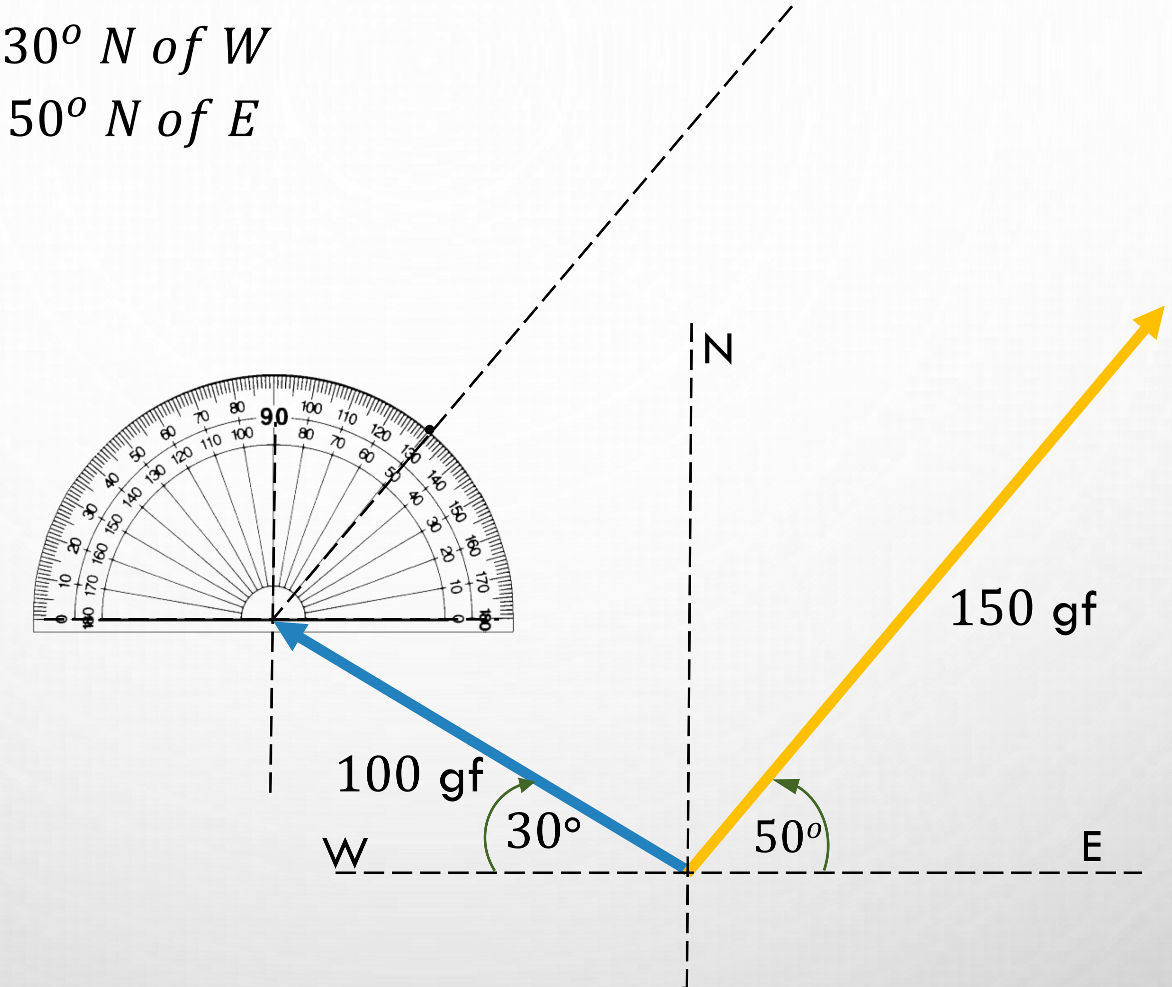

Two forces

(Drawn to scale)

• Graphical Method: Parallelogram Method

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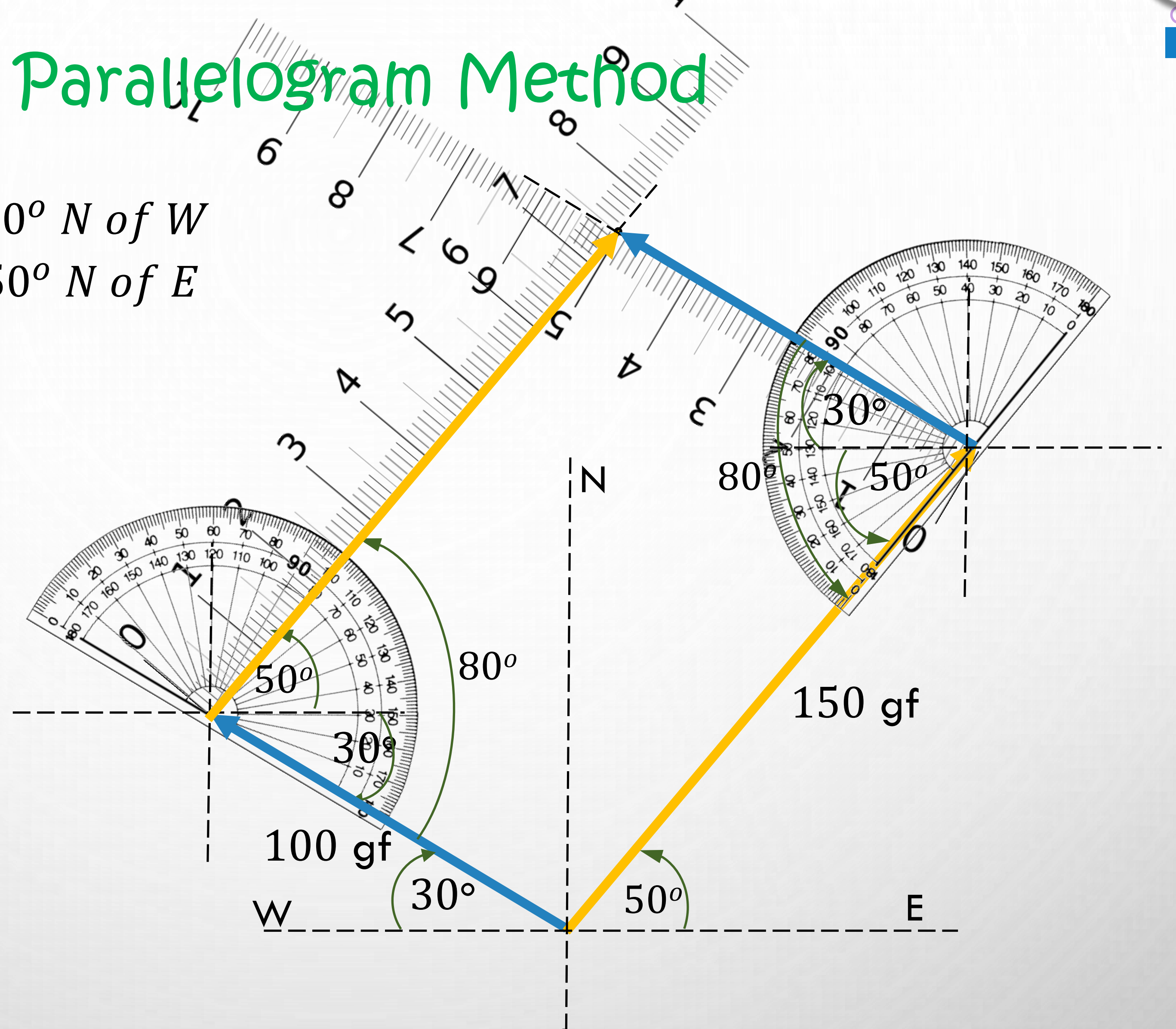


Two forces

(Drawn to scale)

- **Graphical Method:** Parallelogram Method

Suppose Given:

$\vec{F}_1 = 100\ gf$ Direction: 30^o N of W

$\vec{F}_2 = 150\ gf$ Direction: 50^o N of E

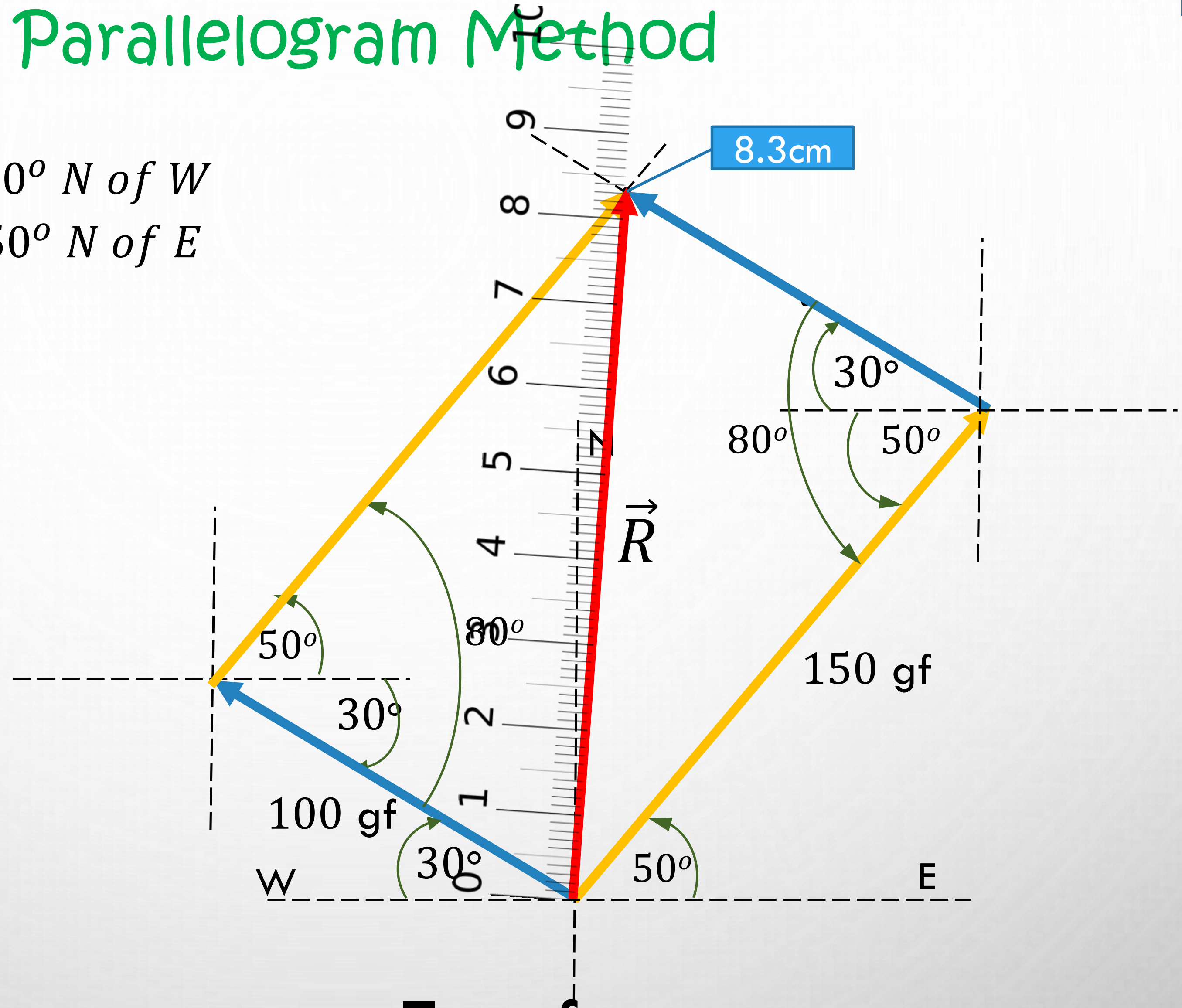
Scale $1cm : 20\ gf$

$100\ gf \times \frac{1cm}{20\ gf} = 5$ cm

$150\ gf \times \frac{1cm}{20\ gf} = 7.5$ cm

$$\vec{R} = 8.3cm\ x\ \frac{20gf}{1cm}$$

$$= 166\ gf$$



Two forces

(Drawn to scale)

• Graphical Method: Parallelogram Method

Suppose Given:

$\vec{F}_1 = 100\ gf$ *Direction*: 30^o *N of W*

$\vec{F}_2 = 150\ gf$ *Direction*: 50^o *N of E*

Scale $1cm : 20\ gf$

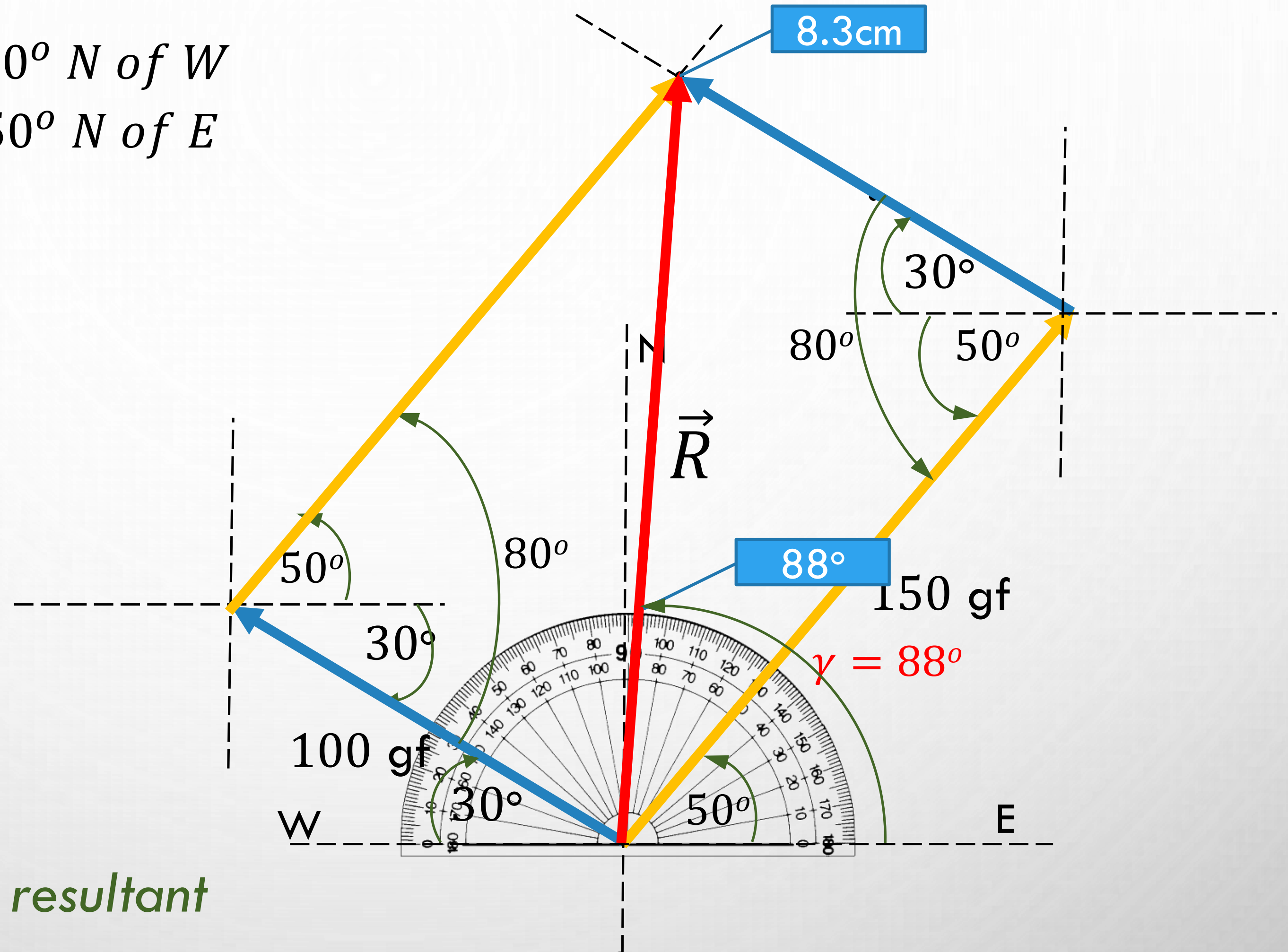
$100\ gf \times \frac{1cm}{20\ gf} = 5$ cm

$150\ gf \times \frac{1cm}{20\ gf} = 7.5$ cm

$$\vec{R} = 8.3cm\ x\ \frac{20gf}{1cm}$$

$$= 166\ gf$$ *magnitude of resultant*

Direction: $\gamma = 88^o$

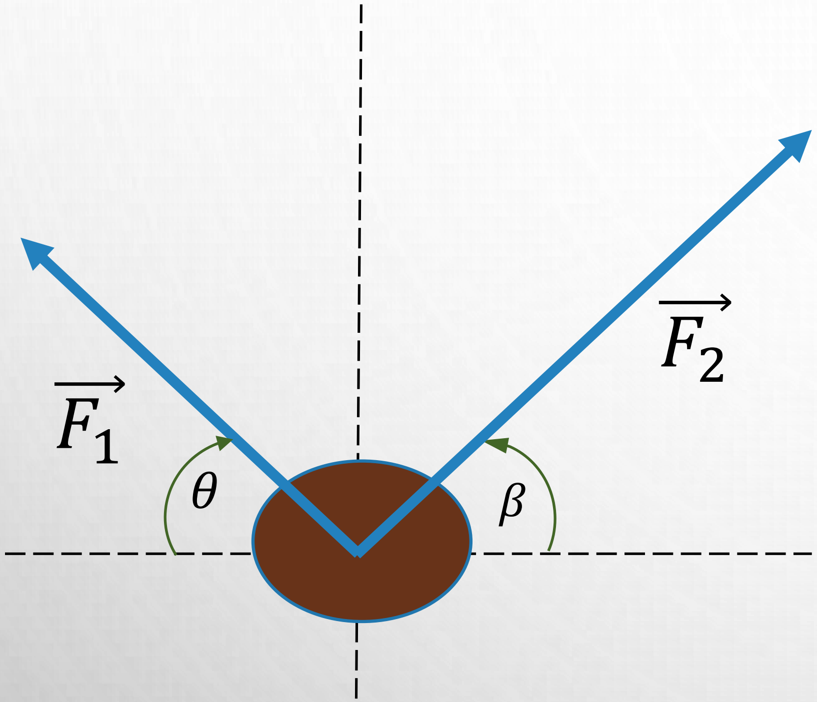


Two forces

(Drawn to scale)

VECTOR ANALYSIS

- Graphical Method Polygon Method

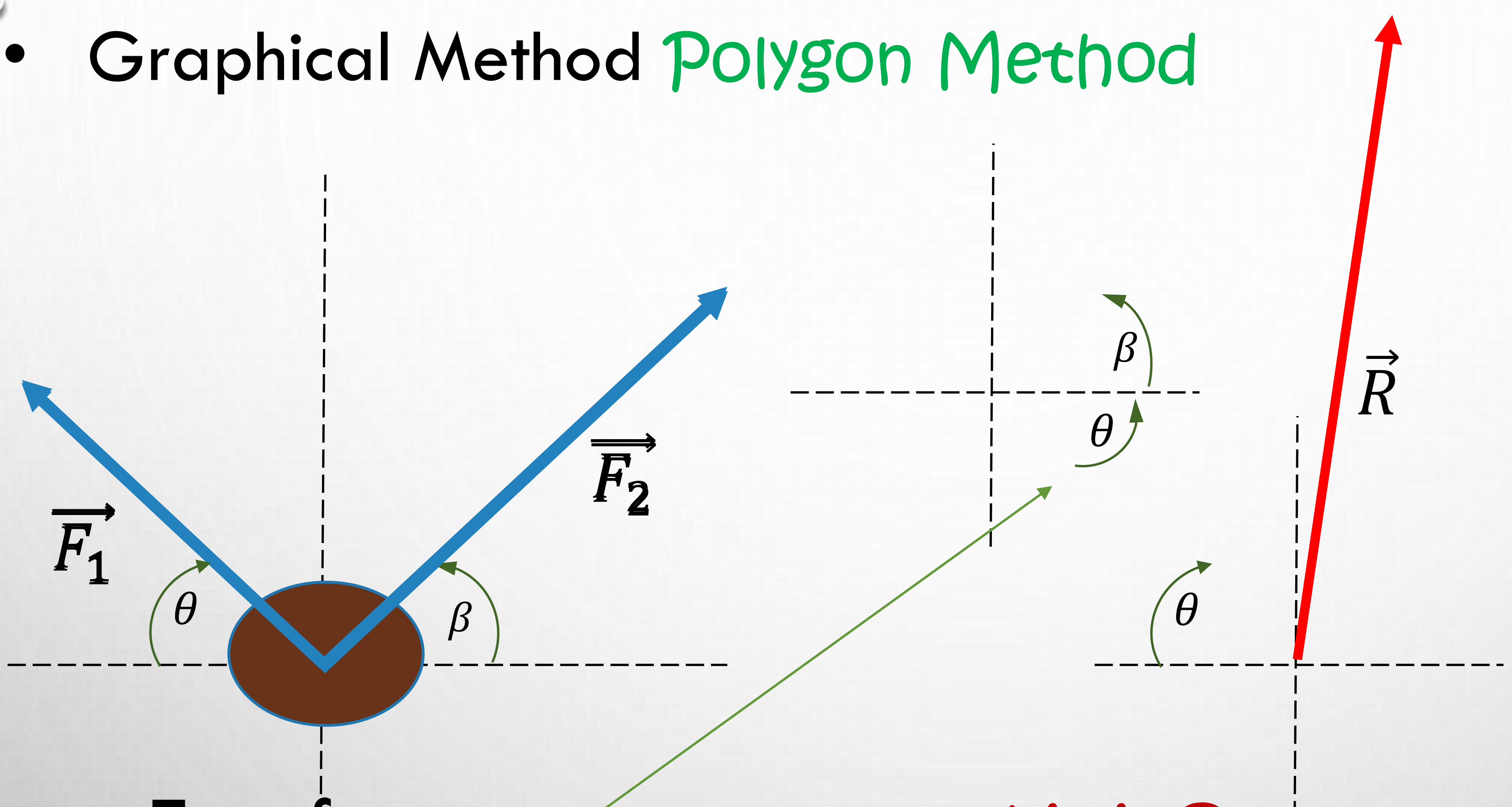


Two forces

(Drawn to scale)

VECTOR ANALYSIS

- Graphical Method Polygon Method



Two forces

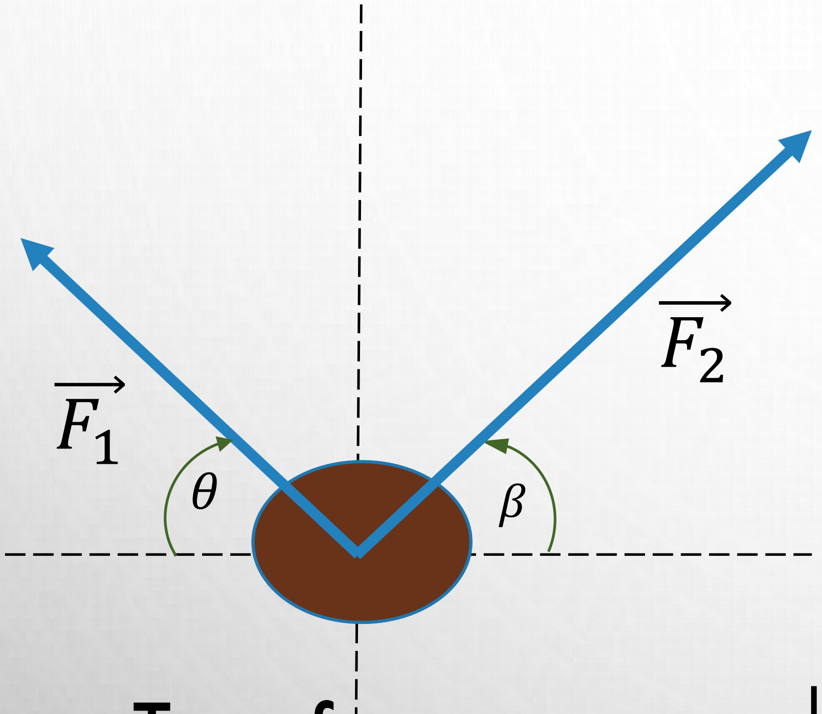
(Drawn to scale)

This is Polygon method

(Drawn to scale)

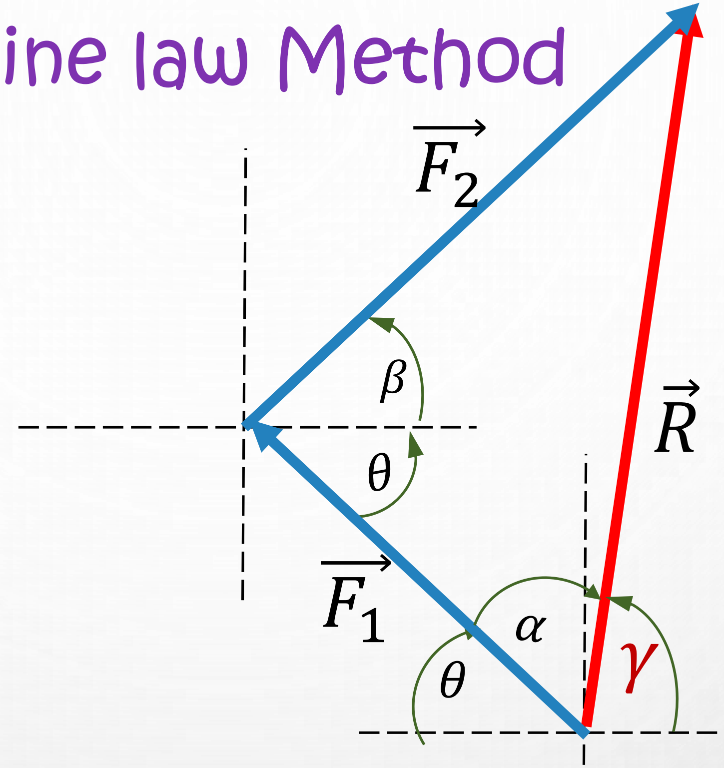
VECTOR ANALYSIS

- Analytical Method Cosine law Method



Two forces

Drawn not to scale



Equivalent polygon of vectors (Drawn not to scale))

Apply law sines to solve for α:

$$\frac{sin\alpha}{F_2} = \frac{\sin(\theta + \beta)}{R}$$

Magnitude of the Resultant R (apply law of cosines)

$$|\vec{R}| \text{ or } R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos(\theta + \beta)}$$

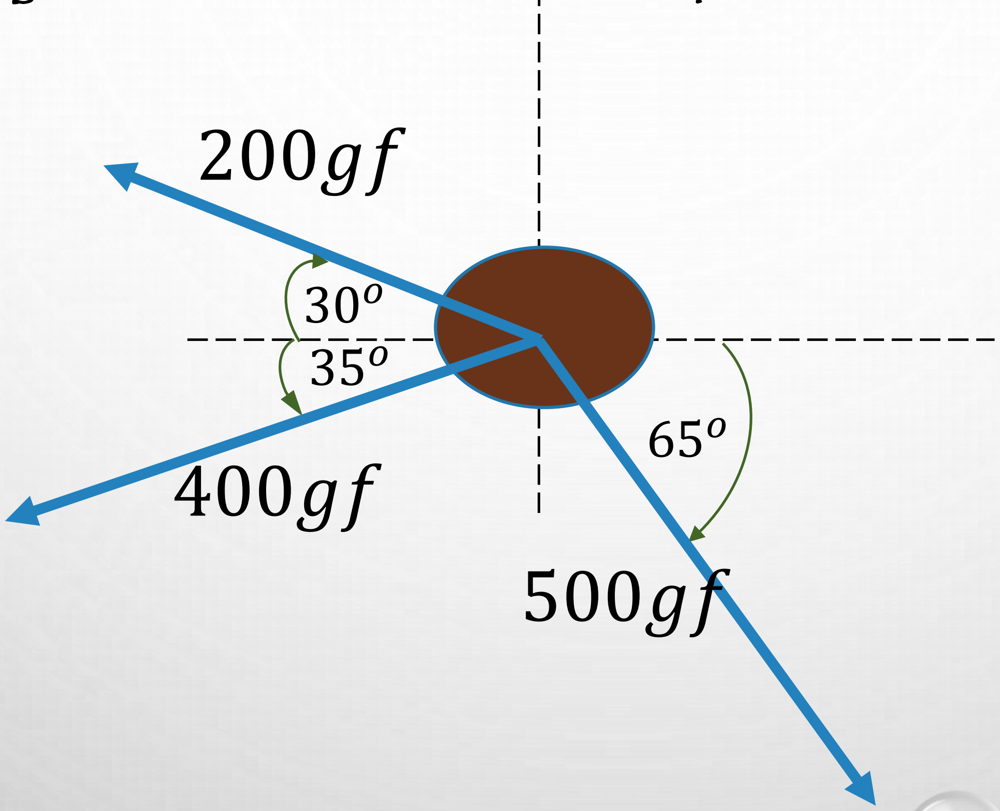
Direction of the Resultant R

$$\gamma = 180^o - \theta - \alpha$$

VECTOR ANALYSIS

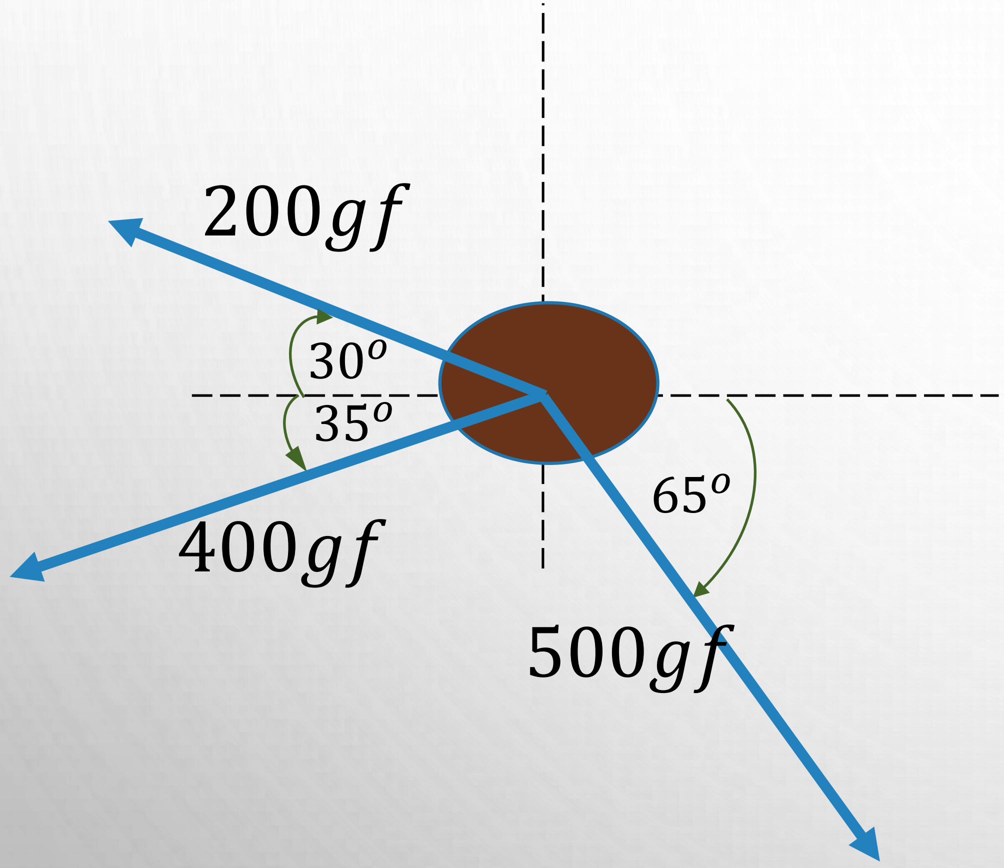
Problem

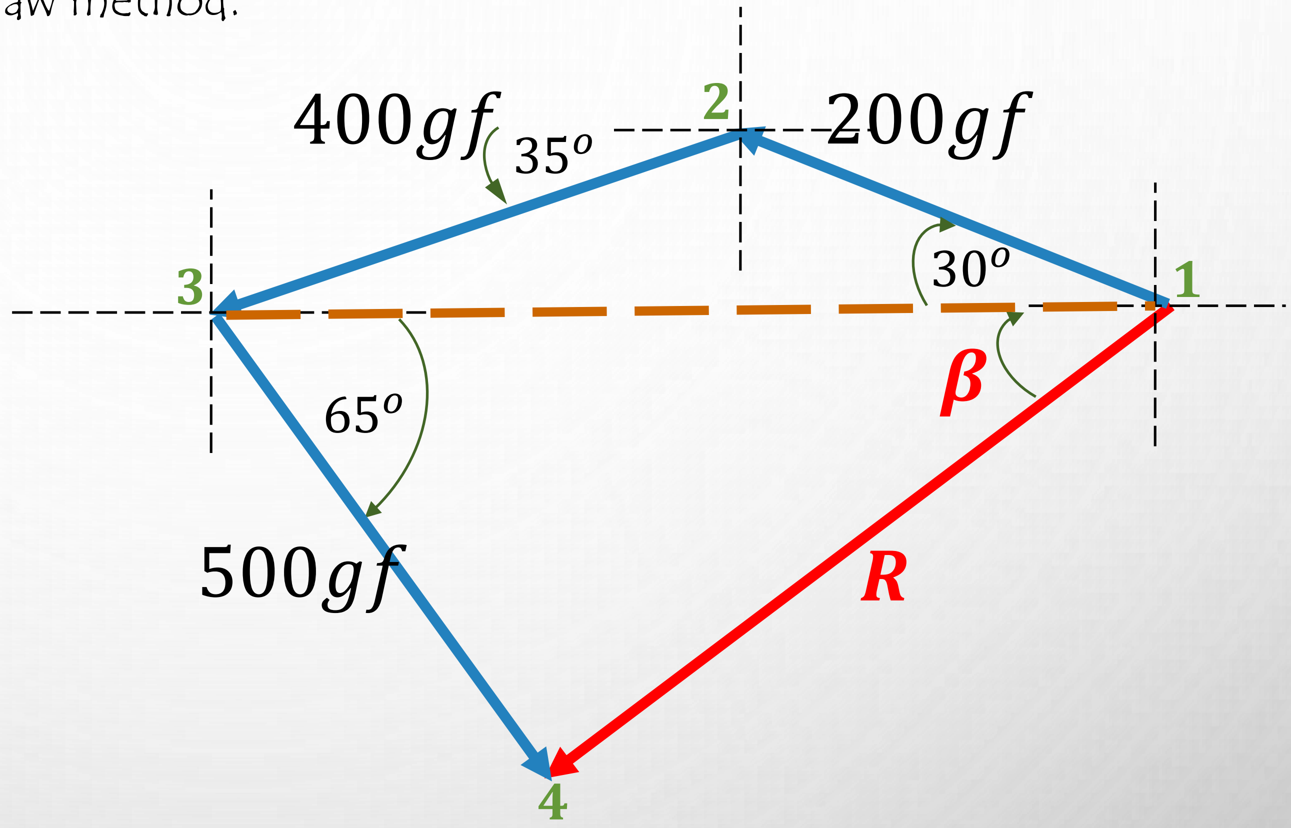
Find the magnitude and direction of resultant using polygon method and component method.



Problem

Find the magnitude and direction of resultant
using polygon method and Cosine law & sine law method.

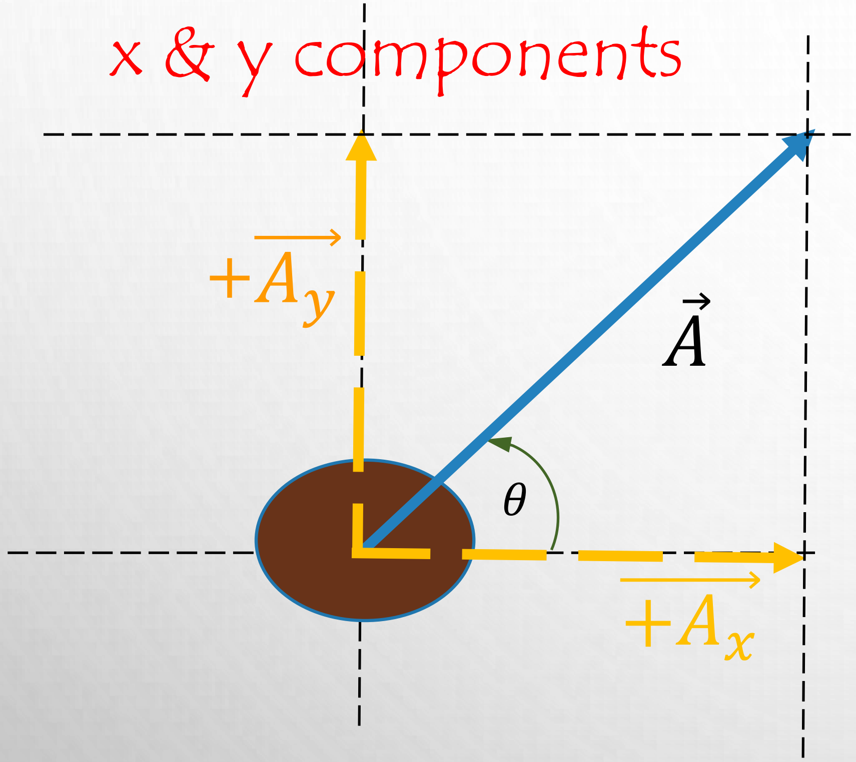




VECTOR ANALYSIS

- Analytical Method Component Method

Vector can be resolve in x & y components

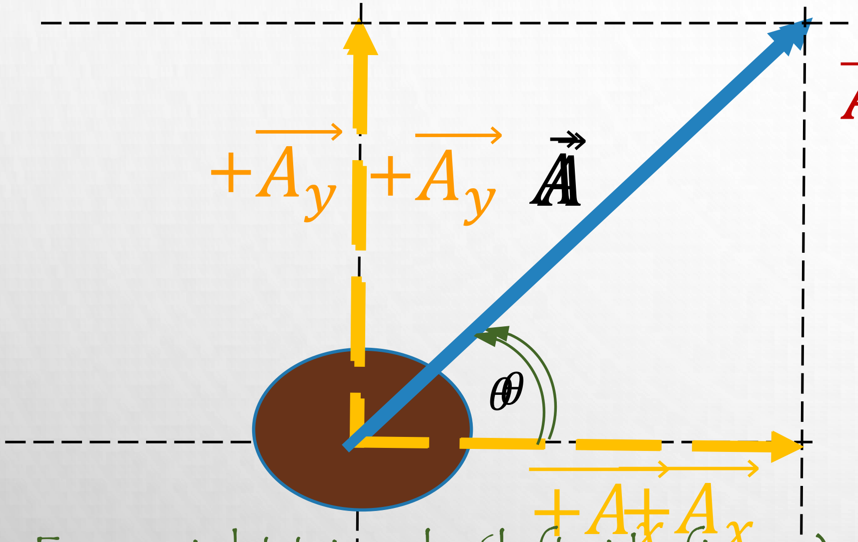


VECTOR ANALYSIS

- Analytical Method Component Method

Vector can be resolve in x & y components

Components and its vector can be formed into a right triangle.



$$\vec{A} = \vec{A}_x + \vec{A}_y$$

From right triangle (left side figure)

$$\cos\theta = \frac{A_x}{A} \quad \Rightarrow \quad A_x = A\cos\theta$$

$$\sin\theta = \frac{A_y}{A} \quad \Rightarrow \quad A_y = A\sin\theta$$

Resolution of vectors

Composition of vectors

$$A = \sqrt{A_x{}^2 + A_y{}^2}$$

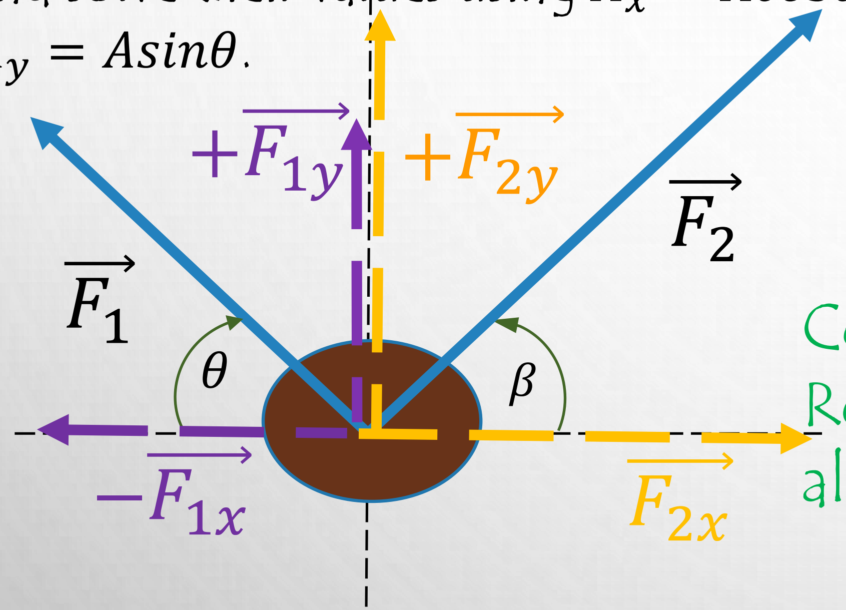
$$\theta = Tan^{-1}\left(\frac{A_y}{A_x}\right)$$

VECTOR ANALYSIS

- Analytical Method Component Method

Steps in Component Method

Step 1: Resolve the vectors into each components and solve their values using $A_x = A\cos\theta$ & $A_y = A\sin\theta$.



Two forces or more

Drawn not to scale

Step 2: Sum up all components along x-axis and all components along y-axis algebraically.

From the Figure (left) we have,

$F_{ix} = F_i\cos\theta$

$$R_x = -F_{1x} + F_{2x} + \ldots$$

$F_{iy} = F_i\sin\theta$

$$R_y = F_{1y} + F_{2y} + \ldots$$

Components of Resultant (R) along x & y axis

Step 3: Calculate the magnitude and direction of the resultant using R_x and R_y.

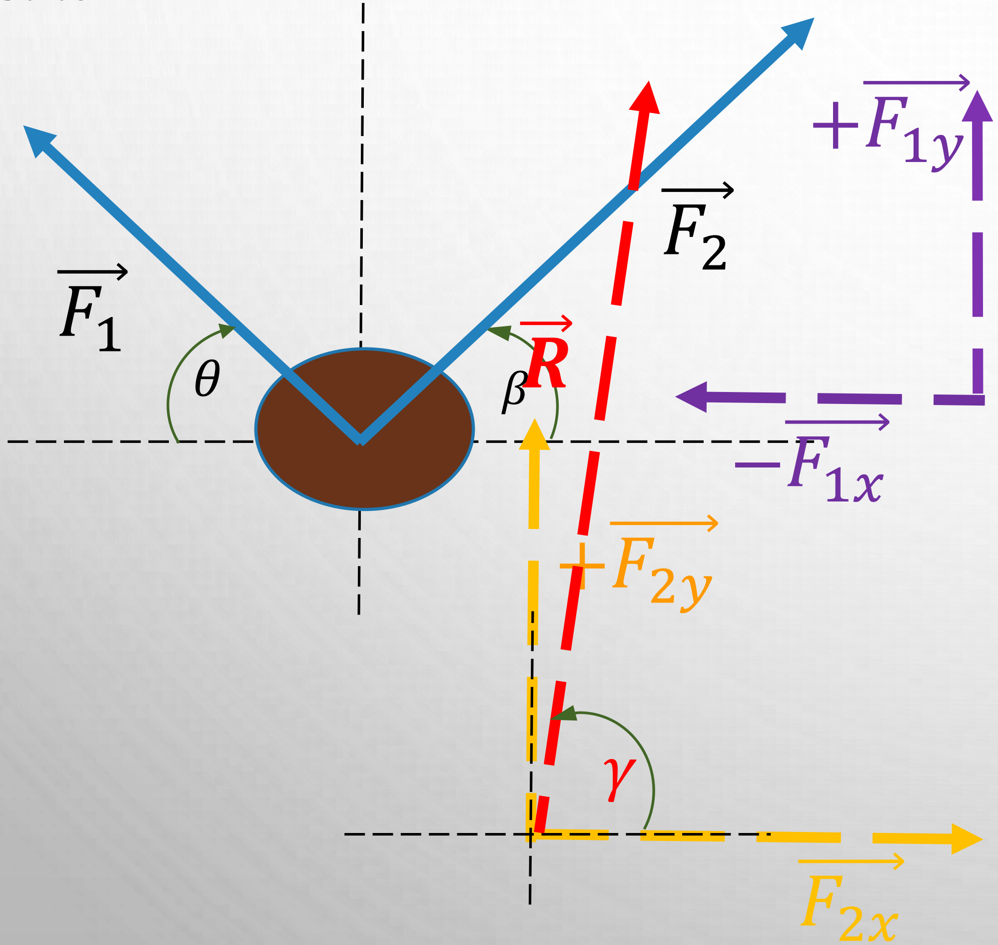
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$ Magnitude of resultant

$$\gamma = Tan^{-1}\left(\frac{R_y}{R_x}\right)$$ direction of resultant

VECTOR ANALYSIS

Steps in Component Method

Step 1: Resolve the vectors into each components and solve their values using $A_x = Acos\theta$ & $A_y = Asin\theta$.



Step 2: Sum up all components along x-axis and all components along y-axis algebraically.

From the Figure (left) we have,

$$R_x = -F_{1x} + F_{2x} + \ldots$$

$$R_y = F_{1y} + F_{2y} + \ldots$$

$$F_{iy} = F_i sin\theta$$

Step 3: Calculate the magnitude and direction of the resultant using R_x and R_y.

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$ Magnitude of resultant

$$\gamma = Tan^{-1}\left(\frac{R_y}{R_x}\right)$$ direction of resultant

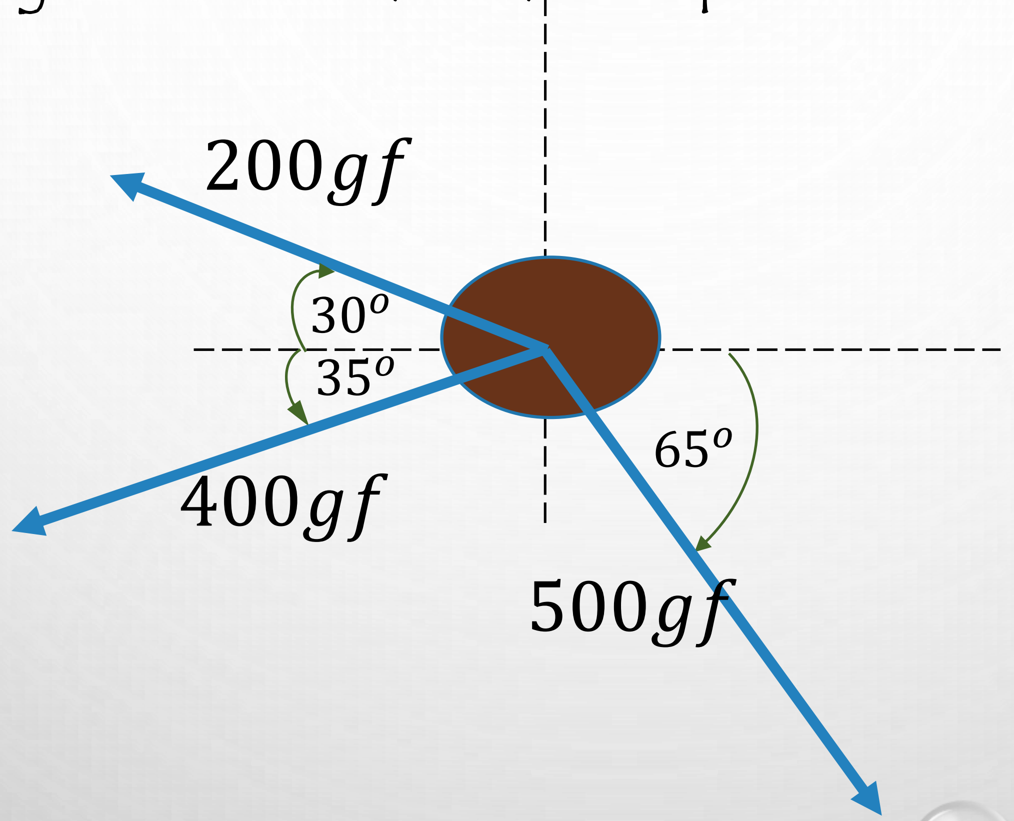
VECTOR ANALYSIS

Force table

VECTOR ANALYSIS

Problem

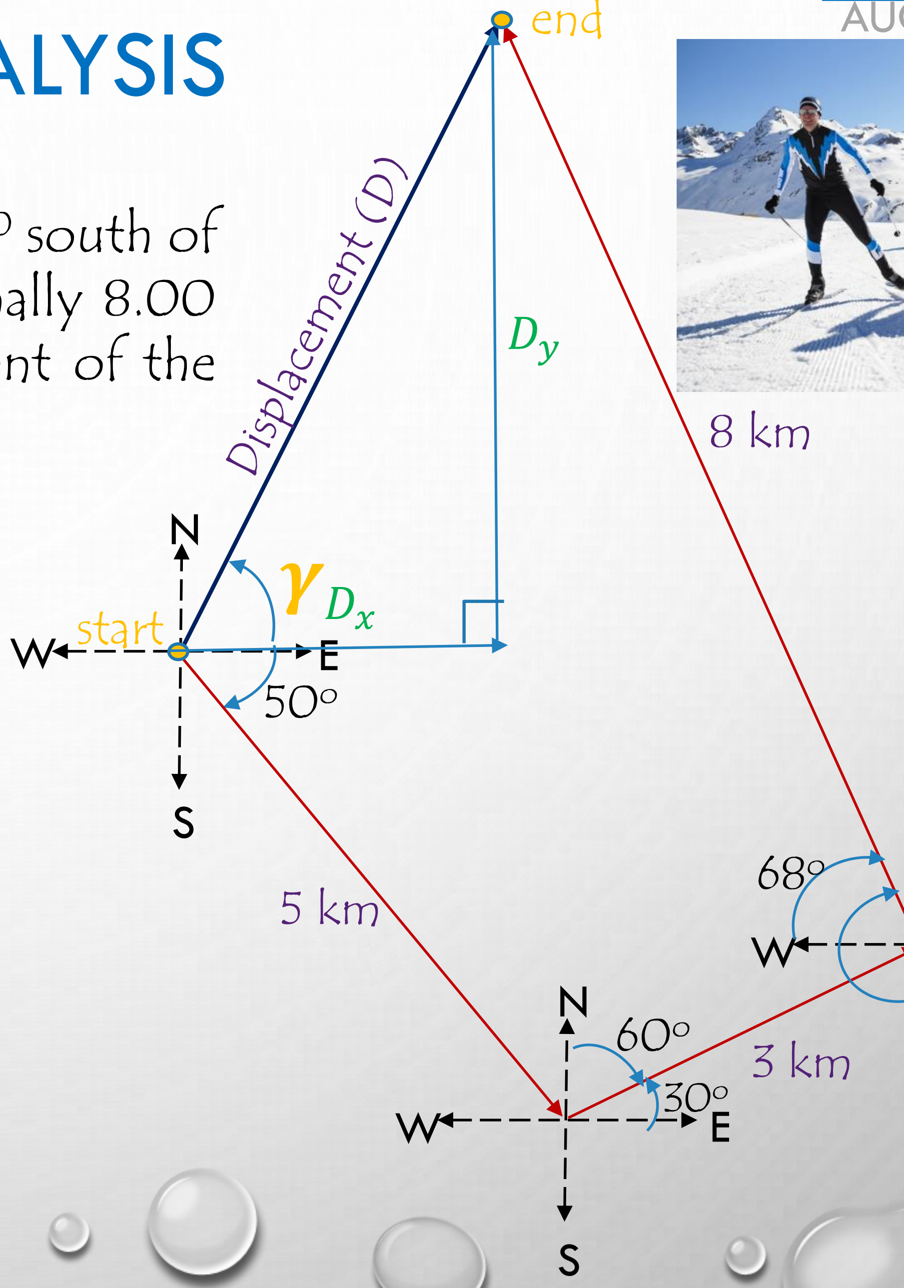
Find the magnitude and direction of resultant using polygon method and component method.



VECTOR ANALYSIS

Application Problem

A cross-country skier skis 5.00 km in the direction 50^o south of east, then 3.00 km in the direction N 60^o E, and finally 8.00 km with bearing angle of 338^o. Find the displacement of the skier.



Solving for the x component of displacement, D_x

$$D_x = +5\cos 50^o + 3\cos 30^o - 8\cos 68^o = 2.82\ km$$

Solving for the y component of displacement, D_y

$$D_y = -5\sin 50^o + 3\sin 30^o + 8\sin 68^o = 5.09\ km$$

Solving for the magnitude and direction of displacement

$$D = \sqrt{2.82^2 + 5.09^2} = 5.82\ km$$

$$\gamma = Tan^{-1}\frac{5.09}{2.82} = 61^o$$ North of East

Department of Mathematics and Physics
College of Arts & Sciences
Visayas State University
Laboratory Activity in General Physics 1

Name:________________ Class time: ______________ Score: /10
Stu. No.:______________ Date Performed:___________
Group No.:______________ School Year:____________ Date Submitted:_______________

VECTOR ANALYSIS

Lab Activity No. 2

Objectives:

The Exercise has the following Objectives:

1. To determine the resultant of several forces using experimental, graphical and analytical methods
2. To compare the various methods in determining the resultant vector

Materials:

Force table, Weight holders, Digital balance, Slotted masses, Spring balance, string, Pulley, Ruler, & Protractor

Procedure

1. Prepare the masses assigned to your group. Weigh them using a digital balance and note your measurement.
2. Anchor the center of the ring to the center of the platform.
3. Attach the pulleys (bobbins) at specific angular directions
4. Attach assigned weigh hangers/ masses (Refer to table 1 and 2) to the strings tied to the center ring of the force table. The string should pass through the pulleys (bobbins)
5. Hooked the spring balance to the free end of the third/ fourth string
6. Pull the spring balance to position the ring at the center. Take the reading and direction of the spring balance or force sensor as your equilibrant
7. Determine the resultant for two forces and three forces using the value of the equilibrant
8. Determine and record the resultant using graphical method. Use parallelogram method for two vectors and polygon method for three vectors
9. Determine and record the resultant using the component method.

Table 1. Adding Two Vectors

Assigned mass	Actual mass	Equilibrant (Force Table)	Resultant (Force Table)	Resultant (Graphical)	Resultant (Component Method)
500g, 30 ° N of E					
1000g, 85 ° S of E					

Table 2. Adding three Vectors

Assigned mass	Actual mass	Equilibrant (Force Table)	Resultant (Force Table)	Resultant (Graphical)	Resultant (Component Method)
500g, 15 ° N of E					
1000g, 70 ° S of W					
300g, 20 ° E of S					

Table 3: % Difference/ Error Measurement

	% difference (experimental and graphical method)		% error (Experimental and analytical method)	
	Magnitude	Direction	Magnitude	Direction
Adding 2 vectors				
Adding 3 Vectors				

Generalization:

1. Compare the different methods in determining the resultant.

2. What is the difference between equilibrant and resultant?

3. Why is it usually difficult to push a sled than to pull it?

Conclusion:

Problems:

1. A car moved 50 km to the North. What is its displacement?

2. Three football players participating simultaneously in a tackle exert the following forces on the ball carrier. 80 g N, 100 g 20° N of E 120 g 35° W of N. What are the magnitude and direction of a single force that would be needed to keep the point at rest.

eNd